First draft

The African Economic Outlook (AEO) model: technical specification and numerical examples

Maurizio Bussolo and Nicolas Gaudemet

April 2003

1 Introduction

The African Economic Outlook (AEO) model presented in this paper was designed to make short to mid term forecasts (up to 3-4 years) on two key variables, output and the rate of inflation, in a simple and standardized way allowing for country comparisons. The model main advantages consist of its simple theoretical structure, parsimonious data requirements, and scalability.

The model explicitly determines aggregate demand through some crucial behavioral equations and a set of exogenous variables, and can then be labeled as "neo-Keynesian". In its simplest version, rather than emphasizing complex macroeconomic relationships, it provides a consistent framework around which available macro data and forecasts can be organized and checked.

The data requirements for the basic version include basic national accounts variables in local currency units (LCU), additional data on main exports, a CPI or other deflators series used to measure past inflation, a series of past RGDP growth rates, some basic parameters on the country economic structure (private propensity to consume, value added composition, etc), and some estimation for the main exogenous variables future paths.

Finally, scalability means that the model can easily be expanded in two dimensions: firstly, new macro behavioral equations can be added to the initial basic version to adapt the model to a particular country context or to study more in detail the determinants of certain macro phenomena; secondly, the parameterization of the model may rely on

available estimations from the econometric literature or can be directly estimated when sufficiently long and dependable series are at hand. Clearly adding behavioral equations and consistently obtaining unbiased and efficient parameter estimates are not unrelated issues, and two-least-squares or other simple econometric techniques may need to be replaced by more sophisticated (and more data and computation intensive) estimation procedures; however, in principle, scaling the model up should be feasible.

Empirical macro-modeling has been criticized on two main grounds: firstly, macro econometric models have been accused of having too shallow micro-foundations, and, secondly, that their "structural" parameters are not policy-invariant and therefore the potential policy advice derived from them can be misleading. Both these lines of criticism are acknowledged, however they do not invalidate all uses of macro-models, which can still provide valuable insights in two respects. First, as already noticed, they provide a consistent accounting framework, allowing to organize data and to study macroeconomic issues in a structured way. Second, even if imperfectly, they offer measurable/quantitative estimates on macro relationships and highlight, at least, signs and order of magnitude of these relationships. In addition and due to the three characteristics described above – namely, theoretical simplicity, scant data needs, and scalability – the macro model described here can be applied to many different countries relatively quickly and thus it allows cross country comparability.

This paper describes in details the model features and includes some numerical examples for a few simulations for Latin American countries. A companion paper describes the actual practical implementation of the model in Microsoft Excel.

The following section illustrates the main economic theory underpinnings of the model and relates it to two widely used macro models, the IMF financial programming (FP) and the WB revised minimum standard model (RMSM). Then section 3 presents all the equations of the model, section 4 shows some numerical applications, and a final section concludes.

2 The AEO model's analytical structure in brief

The AEO model is a short-term demand-driven model for an open economy and it is based upon a few behavioral equations of macroeconomic agents and some structural income and price equations. Its accounting framework can be represented in a matrix format as shown in Figure 1. The matrix is organized as a macro-SAM (Social Accounting Matrix), i.e. expenses are recorded down the columns, receipts are read along the rows and each row total should be equal to the corresponding column total so that each account is balanced and total receipts are matched by total outlays.[1]

Figure 1: Macro accounting framework

	Suppl.	Househ.	Gov.	CapAcc.	ROW	
	1	*2*	*3*	*4*	*5*	
1 Suppliers	IC	C	G	I	E	Demand
2 Households	Y					Income
3 Government		T				Receipts
4 Capital Account		Sh	Sg		Sf	Savings
5 Rest of the World	M					Imports
	Supply	Expend.	Expend.	Investment	ForExchg.	

1) **Y+M = C+G+I+E**	*GNP: Value added + Imports = Consumption + Gov Expenditure + Investment + Exports*
2) **C+T+Sh = Y**	*Domestic Income: C + Direct taxes + Hh Savings = Income*
3) **G+Sg = T**	*Government Budget: G + Gov. Savings = Taxes*
4) **I = Sh+Sg+Sf**	*Investment = Savings (private + public + foreign)*
5) **E + Sf = M**	*Foreign Balance*

At the bottom of the matrix, five accounting identities represents the budget constraints of each account of the matrix. In particular, the AEO model starting point is the first identity, the one sector material balance, where imports *M*, exports *X* and private consumption *C* are endogenously determined and private and public investment as well as public consumption *G* are exogenous.

There are three key macro balances that are considered in the model: the savings-investment balance, the government budget and the foreign exchange balance.

[1] A quite large literature on SAMs now exists and readers interested in more detailed description should start from Pyatt and Round (1985) and the recent CEC, OECD, IMF, UN, and WB revised manual on the System of National Accounts (1993).

Like investment, total savings are exogenous. In the absence of any exogenous change of investment (whether the model is used for simulation or for forecasting), any change in a component of savings must be compensated for by an equivalent change of another component. In particular, the applied model does not formalize explicitly foreign savings (or, in other words, capital inflows and outflows), and government savings are residually calculated as the difference between exogenously set government expenditures and fixed tax rates. These assumptions have the strong implication that excess government demand or large trade deficits do not directly affect the price level. Things would be totally different under a "neoclassical" closure, whereby available savings would determine private investment; a "savings-driven" model of this kind would be more justified for the study of long-run issues.

On the supply side, the model makes the strong assumption of unconstrained capacity as there is neither explicit production function nor any rate of utilization of installed capital. However, via a price-cost loop that links the unitary cost of labor to prices, an implicit short run production function is used to derive an upward sloping aggregate supply (see infra). Furthermore, the capital stock is kept fixed and investment plays the some role as other components of demand: there is no accelerator effect (whether based on expected demand or on profits), unlike other neo-Keynesian models.

On the price side, the model embeds also a quite simple approach. There is a perfect dichotomy between the real sphere and the monetary sphere: money is neutral. There is no rate of interest, unlike in the IS-LM Hicks' story. Monetary emission does not influence prices or exchange rate, which is exogenous. In the spirit of scalability, a money market equilibrium could be added and written as follows:

$$\overline{LG} + LP + e\,R^{w} = M^{s} = M^{d} = k\,P\,Y$$

where money demand is derived, through a Cambridge quantitative equation, from the GDP in value, where internal financing of the public deficit (loans to the government *LG*) is exogenous, and where net external reserves (eR^{w}) are derived from the balance of payments, making loans to the private sector a possible closure variable.

In its simplest version, not only monetary market tensions but also goods and labor market gaps do not weight on the price level (there is no Phillips equation for instance).
The price level is determined as follows. The price of value added, a key component of other prices, marks up the unitary cost of labor. Wages are exogenous and the unitary cost of labor is calculated by inverting a short-run production function where higher production is obtained through increased employment only. With no simultaneous increase in capital, the additional labor is less productive, leading to an increase in the unitary cost of labor. Then particular rigidities of the labor market can be taken into account through the elasticity of production to labor.

2.1 Standard applied macro models compared to the AEO model

The Financial Programming model

The Financial Programming model is an applied macro model first designed in the late 50's and used by the IMF. As such, is in the line with the mandate of the financial institution, which is to ensure a stable world economy in financing temporary balance of payment disequilibria. Economic stability is perceived to be low inflation and sufficient international reserves: the model focuses on monetary factors. Two instruments are privileged to help ensuring a short run stabilization: the nominal exchange rate and the volume of domestic credit.

The FP model is not traditionally used to make previsions. Analysts usually set a target in terms of inflation and balance of payment deficit, then compute, with the model, what nominal exchange rate and domestic credit is needed to reach such a target (hence the name of Financial Programming).

The model relies on simplifying assumptions, for which it has been widely criticized, given the influence of the IMF policy advice in the developing economies.[2] First, one could argue that balance of payment sustainability is not an objective in its own right, and

[2] See for example Taylor 1988, Edwards 1989 or Tarp 1993

the focus on monetary issues leads to the fact that the material balance is not modeled. Second, the choice on the closure – i.e. the exogenous and endogenous variables – depends on economical assumptions that may not reflect the actual economy. For instance, the government deficit and revenue is exogenous, leading the public consumption to adjust. Such an assumption outlines the IMF emphasis on expenditure cuts rather than revenue increase. Furthermore, the choice on the closure may influence the qualitative results of the model. Third, the model relies on neoclassical equations assuming that equilibria solve instantly through flexible prices, without any of the structural rigidity that are crucial in developing countries.

The Revised Minimum Standard Model

Dissimilar to the FP model, the RMSM focuses on mid-long term RGDP growth. As such, it is also referred to as the Growth Programming model. The economy is described with real variables, which allows for a certain degree of disaggregation between different sectors (mining, agriculture, manufacturing, etc.) and different exports, and then for more accuracy than the monetary approach of the FP model.

The model has grown in the 60's out of the so-called "two gaps" literature: given a preset level of growth, what constraints must the savings and the foreign exchange (the two gaps) respect, or in other words, how much investment and external borrowing does the country need? However, the focus on real variables completely occults the monetary side of the economy. Other simplifications are made: there is no public finance, hence no link between the government deficit and the potential inflation (the inflation resulting when the government deficit is financed though money creation). In the usual set of exogenous/endogenous variables, additional capital inflow is assumed to be used for investment and no more for private consumption[3]. Like in the FP model, qualitative results may differ, depending on the choice of the closure.

Furthermore, the supply side is not modeled. There is no rate of utilization of installed capital, assuming that there are no constraints on production capacity. At last, like the FP

[3] See Tarp and Brixen 1996, p. 47

model, the standard RMSM relies on neoclassical equations assuming that equilibria solve instantly though flexible prices, without modeling any of the structural rigidity that are crucial in developing countries.

The AEO model in its simplest version is closer to the RMSM, where growth is driven by demand. One important difference is that, even if the supply side is not explicitly modeled, in the AEO model supply is at least disaggregated between different exogenous variables (like food production, export crops production, mining and other sectors), allowing for increased accuracy. Another important difference with respect to the RMSM is that the modeling effort is made on private consumption rather than investment. In fact investment is exogenous in the AEO model while it is proportional to growth in the RMSM. Furthermore, the AEO model, in its determination of national income, details different sources: public and private wages, income from agriculture and self production and other sources. Finally, one key difference of the AEO model from both the FP model and the RMSM is the treatment of inflation. In these models prices are defined by neoclassical equations and are almost exogenous. In the AEO model however, prices are determined through production costs mark-ups.

3 Main equations of the AEO model

The description of the AEO model is organized around 3 main blocks of equations: the national account block, which defines real GDP from the demand side; the income generation and private consumption block; and the prices determination block As already mentioned, in the basic version of AEO, the supply side and the monetary side are not modeled. Balance of payment is unconstrained and capital flows are not modeled either. Government deficit is unconstrained, however a few equations in a fiscal block help calculate its revenues and expenditures; notice, though, that these latter equations do not influence the result of the model. For convenience, we present first the income account and its link with private consumption, which is central to the model. Then we present the national accounts in volume, and the prices block follows. A final sub-section shows how the government budget is generated in the model.

3.1 Income generation and private consumption

The model decomposes the household income (in value) into five components:

- Food crops income
- Export crops income
- Income from public employment
- Row transfer to households
- Other private income

Representing a significant amount of income in most developing countries, food crops income and export crops income are exogenously estimated by the analyst. Exports crops are a component of the main exports aggregate variable, however producers receive a lower price for their sales than the one used to evaluate international exports. This price differential generates income that goes to intermediaries who deal with the transport and marketing of the farmers' output. Both the domestic producer and the international border prices are estimated exogenously by the analyst. It is important to note that for non agricultural primary exports, normally oil or other mining resources, the producer price is set to zero. The revenues of these non agricultural primary exports are shared between foreign companies and the government (that may receive royalties for instance, which are taken in account in the fiscal account as other revenues). It is then clear that a main exports shock will have different effects on real GDP whether the export is agricultural or non agricultural; more details on these mechanisms are given below in section 4.

Public employment and public wages are exogenously set, leading to an exogenous income from public employment. Row transfers to household (from the government or the rest of the world) are also exogenous.

The last component of income, other private income, is identified with the cost of labor employed to generate the private sector output excluding food and export crops. It is important to note that both formal and informal sector contribute to the private sector output.

More in detail the equation determining other private income is derived from a short term production function as follows. It is assumed that, in the limited forecasting time period, capital stock cannot be increased and that additional output can only be obtained by increasing employment. In addition, employment is not instantly adjusted to its *desired* target level. In a standard Cobb-Douglas function, production can thus be written as:

$$RGDP_{priv} = A \cdot e^{\tau \cdot (year\text{-}year\_base)} \cdot \left(L^*_{priv}\right)^{\varepsilon_L} \quad (1)$$

where $RGDP_{priv}$ stands for non agricultural, both formal and informal, private real GDP, A is a constant L^*_{priv} is non agricultural, both formal and informal, private labor, used to generate this part of the real GDP; the star indicates that it is a targeted employment level; ε_L is a parameter. Formally $RGDP_{priv}$ is derived as total $RGDP$ minus the real public wage bill, minus exports and real food production (which may include self-production):

$$RGDP_{priv} = RGDP - \frac{adj_G}{W_{pub}} G_W - X_{main} - \frac{Yh_{food}}{P_{food}} \quad (2)$$

We can eliminate the constant in (1) using ratios, and then invert (1) to get L as a function of production (notice that the subscripts "–1" indicate a 1 time lag in the variable):

$$\frac{RGDP_{priv}}{RGDP_{priv-1}} = e^{\tau} \cdot \left(\frac{L^*_{priv}}{L^*_{priv-1}} \right)^{\varepsilon_L}$$

$$\frac{L^*_{priv}}{L^*_{priv-1}} = \left(\frac{RGDP_{priv}}{RGDP_{priv-1}} e^{-\tau} \right)^{\frac{1}{\varepsilon_L}}$$

The adjustment process from actual labor toward targeted labor is modeled as follows:

$$\frac{L_{priv}}{L_{priv-1}} = \left(\frac{L_{priv-1}}{L_{priv-2}} \right)^{1-\sigma_Y} \left(\frac{L^*_{priv}}{L^*_{priv-1}} \right)^{\sigma_Y}$$

Writing other private income as the private wages index (W_{priv})[4] multiplied by the labor employed in the private non agricultural sector, we obtain:

[4] One should note that the private wages index should be elaborated with great care, as it must take account for the non agricultural, both formal and informal economy.

$$Yh_{oth} = W_{priv} \cdot L_{priv}$$

then:

$$\frac{Yh_{oth}}{Yh_{oth-1}} = \cdot \frac{W_{priv}}{W_{priv-1}} \cdot \left(\frac{W_{priv-2}}{W_{priv-1}} \cdot \frac{Yh_{oth-1}}{Yh_{oth-2}} \right)^{1-\varsigma_Y} \cdot \left(\frac{RGDP_{priv}}{RGDP_{priv-1}} \cdot e^{-\tau} \right)^{\frac{\varsigma_Y}{\varepsilon_L}}$$

to get finally:

$$Yh_{oth} = Yh_{oth-1} \cdot (1 + gW_{priv})(1 + gW_{priv-1})^{\varsigma_Y - 1} \left(\frac{Yh_{oth-1}}{Yh_{oth-2}} \right)^{1-\varsigma_Y} \left(\frac{RGDP_{priv}}{RGDP_{priv-1}} \cdot e^{-\tau} \right)^{\frac{\varsigma_Y}{\varepsilon_L}} \quad (3)$$

The reader must be aware that an increase in private, non agricultural employment is treated in to symmetric way to a decrease, which fails to take account for specific rigidities of the labor market. Equation (3) is basically transforming any increases of RGDP that are not accounted for in food production, main exports or public sector employment, into *nominal* households income using a private wage index.

3.1.1 From household income to household consumption

Household consumption is determined by disposable household income with the following equation:

$$C_h = \chi_C \cdot C_{h-1} \cdot (1 + gPop) + (1 - \chi_C) \cdot \alpha_C \cdot Y_d^{real} \quad (4)$$

Household consumption is composed of two terms: an hysteresis effect, based on the hypothesis that households partly keep the same pattern of consumption of the precedent year; and a revenue effect: current household consumption is also driven by the current real disposable income – which is the above defined household income in value divided by the consumer price index – in a fixed marginal propensity to consume α_C; in other words, consumption imperfectly adjusts to observed changes in disposable income, sometimes reflecting a smoothing behavior.

It can be noticed that the generation of household consumption in AEO is based on simple behavioral assumptions absent in basic forms of the RMSM (it is not modeled at all in the FP model).

Given its focus on demand, supply-side shocks cannot be directly modeled in the current version. A supply shock such as an increase in food production, due for instance to favorable weather conditions, can only be indirectly accounted for by an exogenous increase in food crops income, which then increases demand and RGDP, through the accounting identity defining RGDP as the sum of demand components (see below). However this final result on RGDP does not necessarily match the initial supply shock, given the absence of explicit supply functions and demand equal supply constraints.

3.2 National Account

Like the RMSM, the AEO model focuses on a one sector material balance, which is defined by the demand side:

$$RGDP = C_h + G + I_h + I_g + \Delta S + \left(X_{main} + X_{oth} + X_{nfs}\right) - \left(M_{ene} + M_{oth} + M_{nfs}\right) \qquad (5)$$

For convenience, we present the equations determining the demand side variables in the same sequence as they appear in equation (5): first consumption, then investment, variation of stocks and external trade.

3.2.1 Endogenous private consumption, exogenous public consumption

We have analyzed in the former sub-section how household consumption is endogenously derived; conversely, public consumption is exogenously set[5]. Normally for the forecasting period, the analyst may use the estimated budget expenses taking into account though that these usually widely differ from their actual realizations.

[5] Quite often there is a gap between national and fiscal accounts, and given that the model uses both sources their consistency become an issue. The model is usually run for a few years on historical data, and this allows calculating an adjustment ratio – between the fiscal account data and national account data – for current expenditure and for investment in values. This historical ratio is then kept constant for the forecasting time period.

Public consumption *G* includes government consumption for marketed goods and the public wage bill, to evaluate (at production cost) non marketed output, such as defense, security or infrastructure.

3.2.2 Exogenous investment

Both private and public investment[6] are given as exogenous. Again, a deep knowledge of the country is required from the analyst in order to make a reasonable assessment of future investment. Since the capital stock is kept fix and the supply side is not modeled, investment affects RGDP directly as a final demand component. Therefore the AEO model is not suited to mid-long term projections, which would have to take account of the effect of investment on future supply.

3.2.3 Exogenous variation of stocks

The variation of stocks is given as exogenous. However, it is not estimated by the analyst, but it is generated from historical data. Variation of stocks is a residual variable used to match the model historical estimates of total real demand with observed RGDP series. During its calibration in the historical period, the model generates all the LHS variables of equation (5) excluding variation of stocks; the sum of these variables is compared to the observed RGDP series and any inconsistencies are netted out in the variation of stock variable. Indeed this variable is a sort of measure of how large is the margin of error made by the model in reproducing historical data.

3.2.4 Exogenous main exports, residual exports depend on world demand and on real exchange rate

A large share of developing countries' exports often comprises just a few commodities, and they normally represent a major source of income. Therefore we have chosen to rely on existing forecasts of main exports that depend on global market conditions or specific

[6] Cf. Note 8

production side issues, rather than letting a demand-driven model generate them endogenously.

The four top main exports are then exogenously fixed, both in volume and in value. Clearly forecasting increasing volumes or increasing prices have different effects on growth rates of RGDP, where, basically, price increases alone affect just the decomposition of nominal GDP, and not the real economy growth rate. As mentioned earlier, a difference is made between agricultural and non agricultural main exports. Agricultural main exports have a local producer price and generate a part of the household income. Non agricultural main exports, such as mining or oil, are almost universally produced by public companies, often associated with a foreign firm. The government receives revenues from these exports and these are recorded among the other revenues account. However in the standard version of the model, no automatic link is made between the main exports and the state revenue – other revenues are indexed on the growth of GDP – and such a link has to be made specifically by the analyst in changing the equation defining the other revenues and indexing them on the growth of non agricultural main exports. The main consequence of these modeling choices is then that agricultural and non-agricultural exports affect RGDP directly through equation (5) but non-agricultural exports do not generate additional multiplier effects unless the government recycle the additional revenues back in the economy by, for instance, increasing its expenditures. These different channels of transmission will be clarified with numerical examples in the next section.

Residual (other or non-traditional) exports are endogenously determined by the model. In addition to an hysteresis effect, they depend on world demand and on a real exchange rate index. Assuming that the elasticities of exports to demand and to real exchange rate are time invariant, and adding the hysteresis effect, we obtain the following equation:

$$X_{oth} = X_{oth-1} \cdot \left(\frac{X_{oth-1}}{X_{oth-2}} \right)^{1-\varsigma_X} \cdot \left(\frac{D_{world}}{D_{world-1}} \right)^{\varepsilon_Y^X \varsigma_X} \cdot \left(\frac{RE_X}{RE_X\_1} \right)^{\varepsilon_P^X \cdot \varsigma_X} \quad (6)$$

Non-factor services exports are supposed to grow with the same growth rate than good exports:

$$X_{nfs} = X_{nfs\,-1} \cdot \frac{X_{oth}}{X_{oth\,-1}} \tag{7}$$

One should note that the AEO model also allows to distinguish tourism receipts as a special export. This is useful for countries where tourism is an important foreign exchange earner.

3.2.5 Oil imports depend on domestic demand only, other imports also depend on real exchange rate

Imports are calculated in a similar way as endogenous other exports. However, the AEO model distinguishes between energy and non energy imports. The difference is that energy imports are supposed to be inelastic to real effective exchange rate or oil prices: if the price of oil gets higher, the domestic demand being unchanged, a country will still import a similar amount; this should reflect the high degree of non substitutability of oil (at least in the short run). Therefore energy imports depend on domestic demand and include an hysteresis effect. Assuming the elasticity of imports to domestic demand to be fixed, and adding an hysteresis effect, we obtain the following equation:

$$M_{ene} = M_{ene-1} \cdot \left(\frac{M_{ene-1}}{M_{ene-2}} \right)^{1-\varsigma_M} \cdot \left(\frac{D_{dom}}{D_{dom-1}} \right)^{\varepsilon_Y^M \cdot \varsigma_M} \tag{8}$$

On the other hand, apart from an hysteresis effect, other imports depend both on domestic demand and the real effective exchange rate. Assuming that the elasticities of imports to domestic demand and to real effective exchange rate are constant, and adding a hysteresis effect, we obtain the following equation:

$$M_{oth} = M_{oth-1} \cdot \left(\frac{M_{oth-1}}{X_{oth-2}} \right)^{1-\varsigma_M} \cdot \left(\frac{D_{dom}}{D_{dom-1}} \right)^{\varepsilon_Y^M \cdot \varsigma_M} \cdot \left(\frac{RE_M}{RE_{M-1}} \right)^{-\varepsilon_P^M \cdot \varsigma_M} \tag{9}$$

As for exports, non-factor services imports are supposed to grow with the same growth rate than good imports:

$$\mathrm{M}_{\mathrm{nfs}} = \mathrm{M}_{\mathrm{nfs}\,-1} \cdot \frac{\mathrm{M}_{\mathrm{oth}}}{\mathrm{M}_{\mathrm{oth}\,-1}} \tag{10}$$

3.3 Prices

Most prices are defined in an intuitive way, as a weighted average of other prices. The price of value added, a key component of other prices, is modeled as a mark up on the unitary cost of private, non-agricultural labor and therefore is linked to the real GDP. This link represents what can be labeled as the aggregate supply of the model and it is the crucial mechanism determining inflation: as demand increases, upward pressure on prices builds up due to diluted labor to capital ratios.

3.3.1 Many prices are exogenous

Public and private wages, food producer domestic price index, public utilities price index are calculated with their growth rate given as an exogenous data. They need to be estimated carefully as inflation and real GDP growth depend on them.

3.3.2 External trade prices

External trade prices are almost exogenous too. Non agricultural export and non energy import price indexes are computed from the weighted Euro and non Euro inflation calculated in local currency, and corrected in the case of imports by the evolution of the tariff rate. This is possible because we have assumed that the shares of exports and imports to the Euro and non Euro zones are constant from the base year to the current year. Formally:

$$P_X = \sigma_X^{EUR} \cdot \frac{EUR}{EUR\_base} WP^{EUR} + \sigma_X^{ROW} \cdot \frac{USD}{USD\_base} WP^{\mathrm{ROW}} \tag{11}$$

$$P_M^{oth} = \left(\sigma_M^{EUR} \cdot \frac{EUR}{EUR\_base} WP^{EUR} + \sigma_M^{ROW} \cdot \frac{USD}{USD\_base} WP^{ROW} \right) \cdot \frac{1 + r_{tax}^{tar}}{1 + r_{tax\_base}^{tar}} \tag{12}$$

where *WP* represents international prices, EUR and USD are the nominal exchange rates with respect to the local currency, r^{tar} is the average tariff level and the σ's are the shares that sum to 1.[7] The ratios of non agricultural exports price index to domestic price index and non energy imports price index to domestic price index are used to calculate two effective exchange rates for exports and imports, which help determine the evolution of the volume of non main exports and non oil imports.

The oil imports price index is computed from the world price of oil in local currency and corrected by the evolution of the tariff rate.

$$P_M^{ene} = WP_M^{ene} \cdot \frac{USD}{USD\_base} \cdot \frac{1+r_{tax}^{tar}}{1+r_{tax\_base}^{tar}} \qquad (13)$$

3.3.3 The price of value added reflects the real GDP level

The price of non agricultural value added is central to our model. We assume that it marks up the targeted unitary cost of labor[8]:

$$P_{va} = B\frac{W_{priv} \cdot L^*_{priv}}{RGDP_{priv}}$$

where *B* is the constant mark up. We have seen that $\mathrm{RGDP}_{\mathrm{priv}} = \mathrm{A} \cdot \mathrm{e}^{\tau \cdot (\text{year-year\_base})} \cdot \left(\mathrm{L}^*_{\mathrm{priv}}\right)^{\varepsilon_L}$

We get:

$$P_{va} = B \cdot \frac{W_{priv}}{RGDP_{priv}} \left(A^{-1} \cdot e^{-\tau \cdot (y\text{-}y\_base)} \cdot RGDP_{priv}\right)^{\frac{1}{\varepsilon_L}}$$

$$= A^{-\frac{1}{\varepsilon_L}} \cdot B \cdot W_{priv} \cdot e^{-\frac{\tau \cdot (y\text{-}y\_base)}{\varepsilon_L}} \left(RGDP_{priv}\right)^{\frac{1}{\varepsilon_L}-1}$$

Setting the prices to be 1 in the base year, we can eliminate A and B:

[7] A strong assumption is made here: terms of trade effects are almost completely ruled out given that the same price indexes are used to derive import and export prices.

[8] This equation is inspired by a structuralist view, and as such represent a major difference between the STOPM model and the neoclassical FP model and RMSM. Cf. also Note 6

$$1 = A^{-\frac{1}{\varepsilon_L}} \cdot B \cdot \left(RGDP_{priv\_base}\right)^{-\frac{1}{\varepsilon_L}}$$

We obtain:

$$P_{va} = W_{priv} \cdot e^{-\frac{\tau \cdot (y - y\_base)}{\varepsilon_L}} \cdot \left(\frac{RGDP_{priv}}{RGDP_{priv\_base}} \right)^{\frac{1}{\varepsilon_L} - 1} \quad (14)$$

An important difference between equations (14) and (3) should be highlighted: no hysteresis effect is included in equation (14). It is assumed that employers setting the price of value added anticipate the targeted employment level and not the actual, more slowly adjusting, employment level. This leads to the following mechanism: when reacting to increased demand, producers hire more people and do not immediately expand their capital stock. New employees are less productive and the unitary cost of labor goes up. Producers pass on to prices these higher costs with no lags (no hysteresis) in order to keep their margins constant, leading to inflationary pressures. In this sense, the equation defining the price of value added plays the same role of an aggregate supply curve, which helps determining how much prices increase when the demand curve shifts to the right.

3.3.4 Other domestic prices are weighted averages of exogenous prices and the price of value added

Other domestic prices are calculated as weighted sums of the previous ones. In fact, the domestic price index depends on value added and intermediates price indexes:

$$P_d = \left[\sigma_{\text{int}} \cdot P_{\text{int}} + \left(1 - \sigma_{\text{int}}\right) \cdot P_{va}\right] \cdot \frac{1 + r_{tax}^{ind}}{1 + r_{tax\_base}^{ind}} \quad (15)$$

The intermediate price index depends on the domestic price index as well as the energy and non energy imports price indexes:

$$P_{int} = \sigma_{int}^{dom} \cdot P_d + \sigma_M^{oil,int} \cdot P_M^{ene} + \sigma_M^{noil,int} \cdot P_M^{oth} \quad (16)$$

The investment goods price index depends on the domestic price index and the non energy imports price index:

$$P_{inv} = \sigma_M^{inv} \cdot P_M^{oth} + \left(1\text{-}\sigma_M^{inv}\right)P_d \quad (17)$$

The consumer price index depends on the food producer price index corrected by a domestic margin, on the domestic price index, on the non energy imports price index corrected by a margin on imports, and on the public utilities price index:

$$CPI = \left(1 + Marg_D\right)\left(\sigma_C^{dom,food} \cdot P_{food} + \sigma_C^{dom,oth} \cdot P_d + \sigma_C^M \cdot \left(\frac{1+\mu_M}{1+\mu_M\_base} \right) \cdot P_M^{oth} + \sigma_C^{pub} \cdot P_{pub} \right) \quad (18)$$

In this latter equation, $Marg_D$ represents a domestic margin and it is exogenous. However, as the variation of stocks, it does not need to be estimated by the analyst and is generated with historical data. It plays the role of a residual variable that makes historical inflation levels consistent with the AEO model framework.

3.4 Fiscal Account

The fiscal account has been modeled in order to generate the government deficit/surplus. However, as mentioned earlier, this deficit is unconstrained and the equations of the fiscal account do not influence the results of the model. In particular, this means that potential links between government deficit and inflation are neglected.

3.4.1 Exogenous public expenditure

Interest payments on debt and net lending are exogenous. They do not even need to be entered to run the model, as they intervene in the fiscal balance only, which is unconstrained.

Government consumption and transfers to household are computed in value with an exogenous volume growth rate of public expenditure and, respectively, intermediates price inflation and consumer price inflation. Public wage bill is calculated with an exogenous growth rate of public employment and an exogenous public wage growth rate. These expenditure need to be estimated carefully because they enter directly real GDP. Transfers to household are part of the households income. We will see in detail how these government instruments affect the economy – according to the AEO model – in section 4.

3.4.2 Endogenous tax revenues with exogenous tax rates

Indirect and direct tax rates, as well as tariff rate are exogenous, although they do not need to be entered by the analyst. They are automatically calculated during the historical years and it is assumed that they will not change during the forecast period.

Indirect tax revenues are calculated on GDP at factor costs. Direct taxes revenues are calculated on government wage bill and private, non agricultural, formal and informal income. This is an approximation because there should not be a direct tax on the informal sector, but we have no way to estimate the size of the informal sector. However, the direct tax rate calculated in the last historical year also applies on private non agricultural informal sector. Trade taxes revenues are calculated on goods imports.

Other taxes revenues and other revenues are supposed to grow at the same rate than the GDP. The analyst must change the equations in order to have them reflect the evolution of main exports revenues, if these commodities are produced by state owned companies or if the state receives royalties on them.

3.4.3 Exogenous grants

Grants are exogenous. Like interest payments, they do not need to be specified to run the model, as they intervene in the fiscal balance only, which is unconstrained.

4 Understanding the AEO model through a multiplier analysis

A multiplier analysis is a useful tool to make clear the underlying assumptions about macroeconomic causality in a model. Calculating a few multipliers help analyze the channels, though which a small change in an exogenous variable translates into a new economic equilibrium. In the mean time, it points out the crucial simplifications of the model and help draw limits of its use. Furthermore, a multiplier analysis gives both qualitative and quantitative information, and allows for country comparisons as it is

shown in a numerical example. The caveat discussed in the introduction section still applies here, and one should not derive strict policy advice from the estimated magnitude of these multipliers.

4.1 A note on the multiplier analysis (1)

A simple way to analytically derive "multipliers" consists of totally differentiating the system. Assume our model has the following simplified structure:

$$Y = C + I + G + X - M$$
$$Yd = Yh_{oth} + Y_{agr} + Y_{pub\_wage}$$
$$Yh_{oth} = kY^{\gamma}$$
$$C = \alpha_C \frac{Yd}{p} + C_0$$
$$M = \alpha_M Y^{\mu}$$

where:

Y is RGDP, Yh_{oth} other private incomes, C private consumption, Y_d disposable income, M imports, G government consumption, and p the price of consumption.

Rewriting the RGDP definition and differentiating, for a shock dG on public expenditures we get:

$$Y = \alpha_C \frac{Yh_{oth} + Y_{agr} + Y_{pub\_wage}}{p} + C_0 + I + G + X - \alpha_M Y^{\mu}$$

and

$$dY = dC + dG - dM$$
$$dY = \frac{1}{1 - \alpha_C \frac{k\gamma Y^{\gamma-1}}{p} + \alpha_M \mu Y^{\mu-1}} dG$$

Clearly this multiplier will be larger the larger is the propensity to consume (α_C) and the smaller the leakage through imports (α_M), a standard result, which is going to be reproduced in the full version of the model. It should also be noticed that, given the non-linearity of the model, the endogenous variable Y is on the RHS definition of the multiplier, so that its precision decreases with the magnitude of the initial shock. Besides in the full version of the model, also prices are endogenous variables, so that total

differentials used to calculate multipliers have more complicated expressions and are still a linear approximation of a non-linear system.

4.2 A note on the multiplier analysis (2)

We point out in this subsection that all the multipliers of the full version of the AEO model have the same structure. In particular, a common effect, interpreted as the effect of real GDP on itself, is present in all the experiments of a small change in an exogenous variable. In fact, to analyze an experiment in a macro economic model similar to AEO. one can always obtain, in writing and expanding the definition of the real GDP from the demand side, a equation of the form[9]:

$$\partial RGDP = A \cdot \partial RGDP + B \cdot \partial ExV$$

$$\partial RGDP = \frac{B}{1-A} \cdot \partial ExV$$

where *ExV* stands for the exogenous variable, of which we want to analyze the impact on real GDP, *B* measures the change on real GDP due to the variation of the exogenous variable, and *A* measures the endogenous change on real GDP. Clearly, this formula holds when the endogenous effect is not too big, that is $A<1$. While the *B* parameter depends on which specific exogenous variable is considered, *A* does not, and it is common for all possible shocks. We can then calculate it as follows:

$A = K_2 (1-K_3) - K_1K_4$

where:

[9] To get this equation, totally differentiate the system of the n equations defining the n endogenous variables. The material balance gives ∂RGDP in function of all the differentiates of the other endogenous variables and *∂ExV*. The n-1 other equations define a square linear system of n-1 variables (the differentiated endogenous variables excluding *∂RGDP*) where *∂RGDP* and *∂ExV* can be considered as parameters of the system. One can solve the system and get the n-1 differentiated endogenous variables as linear functions of *∂RGDP* and *∂ExV*. Substituting in the material balance equation gives $\partial RGDP = A\ \partial RGDP + B\ \partial ExV$.

$$K_1 = \frac{\partial P_d}{\partial RGDP} = \frac{\left(1\text{-}\sigma_{\text{int}}\right)\cdot\left(1+r_{tax}^{ind}\right)}{1+r_{tax}^{ind}\_base\text{-}\sigma_{\text{int}}\cdot\sigma_{\text{int}}^{dom}\cdot\left(1+r_{tax}^{ind}\right)}\cdot\frac{1\text{-}\varepsilon_L}{\varepsilon_L}\cdot\frac{P_{va}}{RGDP_{priv}}$$

$$K_2 = \frac{\partial C_h}{\partial RGDP} = \frac{K_5}{CPI}\cdot\left(\left(1-r_{tax}^{dir}\right)\cdot\frac{\varsigma_Y}{\varepsilon_L}\cdot\frac{Yh_{oth}}{RGDP_{priv}} - \left(1+m\arg_D\right)\cdot\sigma_C^{dom,\,oth}\cdot\left(Y_d^{real} - \frac{Tr_g^h}{CPI}\right)\cdot K_1\right)$$

$$K_3 = \frac{\partial M}{\partial D_{dom}} = \varepsilon_Y^M\cdot\varsigma_M\cdot\frac{M_{ene}+M_{oth}+M_{nfs}}{D_{dom}}$$

$$K_4 = \frac{\partial\left(M-X\right)}{\partial P_d} = \frac{\varepsilon_P^X\cdot\varsigma_X\cdot\left(X_{oth}+X_{nfs}\right)+\varepsilon_P^M\cdot\varsigma_M\cdot\left(M_{oth}+M_{nfs}\right)}{P_d}$$

$$K_5 = \left(1-\chi_C\right)\cdot\alpha_C$$

(See the annexes for the detailed calculus).

K_1 is the endogenous variation of the domestic price index due to an endogenous variation of the real GDP.

K_2 is the endogenous variation of household consumption, due to an endogenous variation of the real GDP.

K_3 is the endogenous variation in imports due to an endogenous variation in domestic demand.

K_4 is the endogenous variation in the trade deficit due to an endogenous variation in the domestic price index.

K_5 is the propensity to consume, corrected by an hysteresis effect.

The expression $A = K_2\,(1\text{-}K_3) - K_1K_4$ can then be interpreted as the effect on RGDP due to an endogenous variation of household consumption (K_2), reduced by a leak through imports ($1\text{-}K_3$) and by a variation in the trade deficit due to a competitiveness effect triggered by an endogenous change in domestic prices ($-K_1K_4$).

The numerical values of the K constants depend on the structure of the country; however, their signs can be established as follows:

$K_1>0$, meaning that a real GDP growth leads to an inflation of domestic prices. It is not surprising to see that K_1 depends on the price of value added. In fact, an exogenous demand increase requires production to go up, and as explained above, this leads to inflationary pressures. Higher prices then reduce the ex-post real GDP through decreased nominal incomes and a reduction in household consumption, and loss of competitiveness leading to an increased trade deficit.

$0<K_3<1$, an increase in domestic demand will be always partly dissipated through imports: an increase in domestic demand leads to an increase in imports.

$K_4>0$, an increase in domestic prices will always increase the trade deficit through a loss of competitiveness. Non-main exports, as well as non-energy imports, directly depend in the model on real effective exchange rates, thus on domestic prices.

We cannot predict the sign of K_2, meaning that the effect of real GDP growth on household consumption is ambiguous. Household consumption is a linear function of real household income, thus it varies according to two opposite effects: on household nominal income, and on prices. An ex-ante real GDP increase results in higher nominal incomes and higher domestic prices, and the final effect on real household income is ambiguous. So an ex-ante increase in real GDP can be either amplified or amortized by consumption. It is straightforward to see, though, that the larger is the endogenous effect of a real GDP change on consumption, i.e. the larger K_2 is, the larger is the multiplier. Due to inaccurate parameterization, the size of K_2 may become problematic, in the sense that the parameter A gets too close to 1. In these cases, the model would produce unrealistic results and when $A>1$, the whole thing collapses. These wrong parameterizations usually depend on wrong combinations of a too low the elasticity of production to labor, ε_L, with respect to the speed of adjustment of private, non agricultural employment income, ς_Y. In this case, a variation of demand translates in a huge variation of employment, which leads the revenue effect in K_2 to be too important compared to the price effect.

The price effect measured by K_1 has always a diminishing effect, through negative effects on both real disposable income and competitiveness. Therefore, with an appropriate set of parameters, the qualitative behavior of the AEO model is satisfying. However, it is based on assumptions that are quite simple and do not necessarily take into account of country specific macro adjustment mechanisms, especially with respect to inflation and money markets equlibria.

4.3 Analytics of a few experiments

We analyze here a few experiments, focusing first on government policy variables[10] such as consumption, investment, transfers to households and public employment[11]. Since primary exports can be crucial for developing economies, we also analyze an experiment of a small change in volume of such an export. At last, because a large share of developing countries are oil and OECD goods importers, we analyze the impact of a change in oil price and of a change in the value of the US dollar compared to the value to the local currency.

4.3.1 Increase in government spending

4.3.1.1 Increase in government investment or consumption

As already mentioned, government investment and consumption have the same effects on RGDP, given that in the short-run investments does not have any effect on supply.
The multiplier are thus:

$$\partial RGDP = \frac{1\text{-}K_3}{1-K_2\cdot(1-K_3)+K_1\cdot K_4}\cdot\partial I_g$$

$$\partial RGDP = \frac{1\text{-}K_3}{1-K_2\cdot(1-K_3)+K_1\cdot K_4}\cdot\partial\left(C_g^{real}+C_g^{other,real}\right)$$

As government investment or consumption enter directly domestic demand, the direct effect of an exogenous change in government investment or consumption is only partly dissipated by a leak through imports (1-K_3). The multiplier is >1 when $K_2 > K_3 + K_2K_3 + K_4$, that is when the endogenous effect of a change in real GDP on household consumption – analyzed in the former subsection – is bigger than the leaks through imports and the degradation of the trade deficit.

The change in consumer prices inflation is given by:

$$\partial \mathrm{CPI} = \left(1+\mathrm{Marg_D}\right)\cdot\sigma_C^{dom,oth}\cdot K_1\cdot\partial \mathrm{RGDP}$$

[10] Multiplier analysis of a change in tax rates or in public wage could be done with a similar method.
[11] It is important to note that in all the experiments involving government, it is assume that its budget is unconstrained, and that it has enough resources to finance increased expenditures.

It is then straightforward to verify that the CPI "multiplier" is >0: an increase in government consumption or investment will always increase consumer prices, through the mentioned channel from increased domestic demand to domestic prices. The size of the effect depends on the structure of the country. The bigger the share of other (non food) goods in consumption is – that is the share of domestic price index in the CPI – the bigger the inflationary pressure from an increase in domestic demand is.

4.3.1.2 Increase in government transfers to households

Government transfers to household generate slightly different effects given that they are a component of households income. The government transfers to households multiplier is given by the formula:

$$\partial RGDP = \frac{(1\text{-}K_3)\cdot K_5}{1-K_2\cdot(1-K_3)+K_1\cdot K_4}\cdot\partial Tr_h^{g,real}$$

The effect of a change of government transfers on household consumption is therefore only measured by the propensity to consume (K_5), since household consumption is assumed to be linear with real disposable income. This change in household consumption is also a change in domestic demand, which is partly dissipated through imports ($1\text{-}K_3$).

The effect of an increase government transfer to households is therefore qualitatively the same than an increase in consumption or investment. The size of the effect is smaller though, and increases with the marginal propensity to consume.

It is important to note that the transfers affect the budget of the government.

The effects on inflation are similar that in the case of government investment or consumption:

$$\partial CPI = (1+M\ \arg_D)\cdot\sigma_C^{dom,oth}\cdot K_1\cdot\partial RGDP$$

Real transfers to household affect real household income, thus affect household consumption linearly.

4.3.2 Increase in government employment

Another potential government policy variable is the level of public employment. Let's suppose that every exogenous variable is fixed except the volume of government

employment, that varies marginally. Public wage index is supposed to be constant, since it is exogenous, then the variation of the volume of public employment $\partial Empl_{pub}$ is given by [12]:

$$\partial Empl_{pub} = \frac{adj_G}{W_{pub}} \cdot \partial G_W$$

We can calculate the multiplier:

$$\partial RGDP = \frac{1 + K_7 \cdot (1 - K_3) + K_1 \cdot K_4}{1 - K_2 \cdot (1 - K_3) + K_1 \cdot K_4} \cdot adj_G \cdot \partial Empl_{pub}$$

$$= \left(1 + \frac{K'_7 \cdot (1 - K_3)}{1 - K_2 \cdot (1 - K_3) + K_1 \cdot K_4} \right) \cdot adj_G \cdot \partial Empl_{pub}$$

where

$$K_7 = \frac{1}{adj_G} \cdot \frac{\partial C_h}{\partial Empl_{pub}} = K'_7 - K_2$$

$$K'_7 = \frac{W_{pub}}{adj_G} \cdot \frac{K_5 \cdot (1 - r_{tax}^{dir})}{CPI}$$

K_7 is the variation of the household consumption due to a small change in the volume of government employment, excluding[13] its endogenous variation due to a change in real GDP. This variation is due both to a change in the household nominal income and a change in consumer prices. Its sign is difficult to predict, and not easy to interpret, because of the fact that it excludes consumption endogenous variation due to a change in real GDP. Nevertheless, the mechanisms at work can be described as follows. Since government wage bill enters the domestic demand as a proxy for non marketed public services, an increase in public employment directly affect real GDP. It also affects household consumption and domestic demand through household income (K_7). The impact on real GDP of a change in domestic demand[14] is partly leaked through increased

[12] It is corrected by an adjusting ratio, in order to be consistent with the national accounts (see above).

[13] Hence the $-K_2$ in K_7. In a similar way, one could write K_1K_4 in the numerator as $(-K_1)(-K_4)$, where $-K_1$ is the variation of domestic price index, excluding its endogenous variation, meaning that the change in domestic prices in purely due in this experiment to the endogenous effect described in the previous note on multiplier analysis.

[14] Government wage bill is a proxy for public service. It had been chosen to exclude it from the domestic demand variable, because raising domestic demand increases imports, and this would lead to the

imports (1-K_3). An increase in public employment doesn't exercise a pressure on prices per se, because pressures come from the price of *private* value added. However, K_1K_4 in the numerator is the positive impact on trade balance through the fictive impact government wage bill would have on domestic prices if there was no endogenous effect due to a change in real GDP. It is balanced by the negative impact on trade balance through the endogenous effect of increased real GDP on domestic price index (K_1K_4) in the denominator.

The numerical value of K_7 depends on the structure of the country, as there is an ambiguous effect on nominal income and prices. Intuitively, K_7 increases with the level of public wages, since a change in public employment will have a bigger impact on household income when public wages are higher. Since $K'_7>0$, the multiplier is >1: the effects of an increase in government employment will always be amplified through higher household income and private consumption. The real GDP will increase because there will be a higher quality public service, and more people will be employed.

The change in consumer prices inflation is given by:

$$\partial CPI = \left(1 + M \arg_D\right) \cdot \sigma_C^{dom,oth} \cdot K_1 \cdot \left(\partial RGDP - adj_G \cdot \partial Empl_{pub}\right)$$

It is interesting to see that an increase in government employment will in general lead to less inflationary pressure than in the case of government consumption, investment, or transfer to household. This point will be verified in the following numerical example. It is due to the fact that an increase in government employment doesn't increase directly the domestic demand[15]: the government wage bill is a proxy for public service, which is by definition not produced by private producers.

implausible result that public service is partly imported, and thus related to the private price of value added. Thus in the model real GDP is the sum of domestic demand, net external trade and public service proxied by government wage bill.

[15] Cf. note 16

4.3.3 Increase in the main exports

Primary exports can be extremely important for some developing economies and it is interesting to briefly study how the model reacts to a change in their volume. Let's suppose then that every exogenous variable is fixed except the volume of the first main export, that varies marginally.

The multiplier is calculated as follows:

$$\partial RGDP = \frac{K_6 \cdot (1 - K_3) + K_1 \cdot K_4 + 1}{1 - K_2 \cdot (1 - K_3) + K_1 \cdot K_4} \cdot \partial X_1$$

$$= \left(1 + \frac{K'_6 \cdot (1 - K_3)}{1 - K_2 \cdot (1 - K_3) + K_1 \cdot K_4} \right) \cdot \partial X_1$$

where

$$K_6 = \frac{\partial C_h}{\partial X_1} = K'_6 - K_2$$

$$K'_6 = \frac{K_5 \cdot P^1_{X,prod}}{WP^1_X\_base \cdot USD\_base \cdot CPI}$$

K_6 is the variation of the household consumption due to a small change in main exports volume, excluding its endogenous variation due to a change in the real GDP[16]. The formulation with K_6' highlights the importance of the producer price. The channels of transmission of a small increase in volume of one main export to RGDP are different whether the export is agricultural or not. In the case of an agricultural export, its increase has a direct effect on RGDP and on households disposable income, whereas this latter effect is absent in case of a non-agricultural export. It is also interesting to note that an inflationary effect is only present in the case of agricultural main export increase through its indirect link on private consumption, an not through an upward sloping agricultural supply function (supply in this case is unconstrained). The numerical value of K_6 depends on the structure of the country and on the producer price of the corresponding export. Due to an ambiguous effect on nominal income and on prices, in can be either >0 or <0. As K'_6>0 however, the multiplier is always >1.

[16] Cf. note 15

The change in consumer prices inflation has the usual expression:

$$\partial CPI = \left(1 + M\ \arg_D\right) \cdot \sigma_C^{dom,oth} \cdot K_1 \cdot \left(\partial RGDP - \partial X_1\right)$$

It is then straightforward to verify that the CPI "multiplier" is >0 for an agricultural export increase in volume: it will always increase consumer prices, through an increased income of domestic agricultural producers, and increased household consumption. It is interesting to see that like an increase in government employment, an increase in the volume of the main export will in general lead to less inflationary pressure than in the case of government consumption, investment, or transfer to household, as it will be verified in the numerical example. This is due to the fact that an increase in the main export volume does not increase directly the domestic demand, but only indirectly through a higher household income and consumption.

As for a non agricultural export, since the producer price is zero the multiplier is 1; household income and consumption remain the same, and the domestic prices are unchanged. As mentioned above, non agricultural export revenues are assumed to increase government revenues and will have RGDP and inflation effects only in the case where the government expands its spending.

4.3.4 Increase in oil price

This sub-section is relevant only for oil importer countries. Let's suppose that every exogenous variable is fixed except the price of oil, that varies marginally.
We can calculate the multiplier:

$$\partial RGDP = \frac{K_9 \cdot \left(1 - K_3\right) - K_8 \cdot K_4}{1 - K_2 \cdot \left(1 - K_3\right) + K_1 \cdot K_4} \cdot \partial P_{oil}$$

where

$$K_8 = \frac{\partial P_d}{\partial P_{oil}} = \frac{\sigma_{\text{int}} \cdot \sigma_M^{oil,\text{int}} \cdot \left(1 + r_{tax}^{ind}\right)}{1 + r_{tax}^{ind} \_base\text{-}\sigma_{\text{int}} \cdot \sigma_{\text{int}}^{dom} \cdot \left(1 + r_{tax}^{ind}\right)} \cdot \frac{P_M^{ene}}{P_{oil}}$$

$$K_9 = \frac{\partial C_h}{\partial P_{oil}} = -\frac{K_5}{CPI}\left(\left(1 + m\ \arg_D\right) \cdot \sigma_C^{dom,\ oth} \cdot \left(Y_d^{real} - \frac{Tr_g^h}{CPI}\right) \cdot K_8\right)$$

K_8 is the variation of the domestic price index due to a small change in oil price, excluding its endogenous variation due to a change in the real GDP. K_8>0, which means that an increase in oil price leads to an increase in domestic prices.

K_9 is the variation of the household consumption due to a small change in oil price, excluding its endogenous variation due to a change in the real GDP. K_9<0, which means that an increase in oil price leads to a decrease in consumption through a decreased real household income.

An increased oil price directly affects the domestic price (K_8), impacting negatively real +GDP through an increased trade deficit (K_8K_4). It also negatively affects private consumption through a reduction of real income (K_9). This negative impact on domestic demand through consumption is partly dissipated through decreased imports. The overall impact on real GDP is negative, through decreased private consumption and increased trade deficit.

It's worth noting that oil price doesn't directly impact volume of oil imports, because of our assumption that oil imports are inelastic to price.

The change in consumer prices inflation is given by the sum of the non endogenous change and the endogenous change due to a change in real GDP:

$$\partial CPI = \left(1 + Marg_D\right) \cdot \sigma_C^{dom,oth} \cdot \left(K_8 \cdot \partial P_{oil} \text{-} K_1 \cdot dRGDP\right)$$

With a standard parameterization, the change in CPI is in the same direction than the change in oil price. However the endogenous effect works in a opposite direction and if it is too large implausible results can be generated. The result of rising oil prices and decreasing inflation indicates though that a strange set of parameters has been used. In fact, we can rewrite the variation of CPI:

$$\partial CPI = \frac{\left(1 + Marg_D\right) \cdot \sigma_C^{dom,oth} \cdot \left(1 - \frac{K_5}{CPI} \cdot \left(1 - r_{tax}^{dir}\right) \cdot \frac{\varsigma_Y}{\varepsilon_L} \cdot \frac{Yh_{oth}}{RGDP_{priv}} \cdot \left(1 - K_3\right)\right) \cdot K_8}{1 - K_2 \cdot \left(1 - K_3\right) + K_1 K_4} \cdot \partial P_{oil}$$

This formulation clarifies that when the elasticity of production to labor (ε_L) is too low compared to the speed of adjustment of private, non agricultural, employment income, this leads to a perverse effect similar to the one described in the case of a too high K_2. Therefore the analyst needs to make sure that the set of parameters is realistic.

Lastly, it should be noted that in the former experiment, while oil price is assumed to change, world price inflation remains constant. In the case where world prices were to vary in the same way than oil price, the negative competitiveness effect would be lower. These prices has to be set exogenously by the analyst.

4.3.5 Increase in nominal exchange rate

The last experiment we analyze is an small change in the value of the local currency, compared to the US dollar. In this case the multiplier is:

$$\partial RGDP = \frac{K_{12} \cdot (1 - K_3) - K_{10} \cdot K_4 - K'_4}{1 - K_2 \cdot (1 - K_3) + K_1 \cdot K_4} \cdot \partial USD$$

where

$$K_{10} = \frac{\partial P_d}{\partial USD} = \frac{\sigma_{\text{int}} \cdot \left(1 + r_{tax}^{ind}\right)}{1 + r_{tax}^{ind}\_base\text{-}\sigma_{\text{int}} \cdot \sigma_{\text{int}}^{dom} \cdot \left(1 + r_{tax}^{ind}\right)} \cdot \left(\sigma_M^{oil,\text{int}} \cdot P_M^{ene} + \sigma_M^{noil,\text{int}} \cdot P_M^{oth}\right)$$

$$K_{11} = \frac{\partial CPI}{\partial USD} = (1 + Marg_D)\left(\cdot \sigma_C^{dom,oth} \cdot K_{10} + \cdot \sigma_C^M \cdot \frac{1 + Marg_M}{1 + Marg_M\_base} \cdot \frac{P_M^{oth}}{USD}\right)$$

$$K_{12} = \frac{\partial C_h}{\partial USD} = -\frac{K_5}{CPI}\left(Y_d^{real} - \frac{Tr_g^h}{CPI}\right) \cdot K_{11} \cdot$$

$$K'_4 = \frac{\partial (M - X)}{\partial USD} = -\frac{\varepsilon_P^X \cdot \varsigma_X \cdot \left(X_{oth} + X_{nfs}\right) + \varepsilon_P^M \cdot \varsigma_M \cdot \left(M_{oth} + M_{nfs}\right)}{USD}$$

K_{10} is the variation of the domestic price index due to a small change in the nominal exchange rate, excluding its endogenous variation due to a change in the real GDP. K_{10}>0, which means that an increase in the nominal exchange rate, or a depreciation of the local currency, leads to an increase in domestic prices. Domestic prices increase because they depend (linearly) on oil prices and other imports prices.

K_{11} is the variation of the consumer price index due to a small change in the nominal exchange rate, excluding its endogenous variation due to a change in the real GDP. K_{11}>0, which means that an increase in the nominal exchange rate, or a depreciation of the local currency, leads to an increase in the consumer price index. Consumer price index is increased both directly through increased import prices and indirectly through increased domestic prices.

K_{12} is the variation of the household consumption due to a small change in the nominal exchange rate, excluding its endogenous variation due to a change in the real GDP. $K_{12}<0$, which means that an increase in the nominal exchange rate, or a depreciation of the local currency, leads to an decrease in consumption through a decreased real household income. Real household income decreases because of an increase in consumer price index.

K'_4 is the variation in the trade deficit due to a small change in the nominal exchange rate, excluding its endogenous variation due to a change in the real GDP. $K'_4<0$, which means that an increase in the nominal exchange rate, or a depreciation of the local currency, leads to an decrease in the trade deficit, since imports get more expensive and exports get cheaper. This is the positive effect of a depreciation on competitiveness.

An increase in the nominal exchange rate, or a depreciation of the local currency, directly affects the real GDP through an decreased trade deficit. This decrease in the trade deficit is compensated by an indirect increase ($K_{10}K_4$) through domestic prices (K_{10}). It negatively affects private consumption through a consumer price inflation that decreases real income of households (K_{12}). This negative impact on domestic demand through consumption is partly dissipated through decreased imports ($1-K_3$).

The overall impact on real GDP is ambiguous. In our numerical example, the multiplier is negative in Ghana and positive in Mozambique. There is a positive impact on trade balance, due to a competitiveness effect, that counters a negative impact on household consumption, due to an increase in consumer prices.

The total change in consumer prices inflation is given by the sum of the non endogenous change and the endogenous change due to a change in real GDP:

$$\partial CPI = K_{11} \cdot dUSD + \left(1 + Marg_D\right) \cdot \sigma_C^{dom,oth} \cdot K_1 \cdot \partial RGDP$$

The story is similar to that of a change in oil price. With a plausible set of parameters, the change in CPI is in the same direction than the change in the nominal exchange rate.

4.4 Country comparison through a numerical example

The following table illustrates the numerical values for a set of different multipliers for two African countries. A few remarks follow.

	Ghana			Mozambique		
	% of RGDP	Multiplier of		% of RGDP	Multiplier of	
		RGDP	CPI		RGDP	CPI
Gov real transfers to hh	0.0	0.491	0.100	3.0	0.872	0.002
Gov. Consumption	1.2	0.511	0.104	2.4	0.889	0.002
Gov. Investment	4.8	0.511	0.104	10.0	0.889	0.002
Main export	10.1	1.000	0.000	2.5	1.189	0.000
Publ Employment	7.2	1.276	0.056	4.4	2.245	0.003
Oil price		-0.013	0.005		-0.026	0.001
USD exchange rate		0.000	0.000		0.000	0.000
		Elasticities of			Elasticities of	
		RGDP	CPI		RGDP	CPI
Gov real transfers to hh		0.000	0.000		0.026	0.001
Gov. Consumption		0.016	0.007		0.025	0.001
Gov. Investment		0.025	0.011		0.089	0.004
Main export		0.101	0.000		0.030	0.000
Publ Employment		0.092	0.009		0.100	0.002
Oil price		-0.031	0.027		-0.016	0.016
USD exchange rate		-0.003	0.342		0.063	0.231

Multipliers are highly dependent on parameters

Increasing the elasticity of imports to world prices of 0.1 in Ghana changes the sign of the USD multiplier. It should also be stressed that a few crucial parameters, such as labor elasticity and adjustment speed of private, non agricultural, employment income, can dramatically affect the model behavior and the size of the multipliers. Due to different scaling, multipliers are not comparable across countries.

Elasticities are dependent on the demand structure of RGDP

A 10% increase in volume of the first main export of Ghana approximately leads to 1 point of higher growth of RGDP, while it is only 0.3 points in Mozambique. This is mainly due to the fact that the first main export counts for 10.1% of RGDP in Ghana and only 2.5% in Mozambique. In the same way, elasticities of government consumption and investment are higher in Mozambique because their share in the RGDP is larger than that of Ghana. However, larger shares are not the only explanation. Should the shares be the same, Mozambique would still be more responsive to changes in government consumption or investment: this can be seen because the multipliers are higher (.89) compared to Ghana (.51). This, in turn. is partly due to a difference in the value of the adjustment speed of income.

As for inflation, one can notice that Ghana seems to be subject to more inflationary pressures due to an increased government consumption or investment than Mozambique. This can be explained by the fact that inflation is driven by the mark up behavior of the value added prices to cost of labor. In Mozambique, elasticity of production to labor has been set to .95 compared to the .6 in Ghana.

Another interesting comment can be made, in noticing that in the AEO model, increasing the main export or public employment (which is a proxy for public service) has better effects – in term of ratio of change of RGDP and CPI – than increasing government consumption, transfers to households, or investment. This is due to the analytical structure of the model whereby an increase of export or of public employment does not encounter supply constraints, so that it does not cause inflationary pressure. Note also that the reaction of consumer prices to an increase of main *non-agricultural* export is 0; and it is a small positive number, for *agricultural* exports. Ghana' s main export is non-agricultural and Mozambique is agricultural.

Elasticities on foreign prices depend on the structure of the CPI

As for elasticities of RGDP to oil price or to the nominal USD exchange rate, it can be noticed that Ghana records smaller values than Mozambique. This can be explained by the fact that Ghana's consumer prices depend to a larger extent on import prices than Mozambique's ones, as shown by its lower elasticities of CPI to oil prices or nominal exchange rate.[17] It is worth noting that in the case of a depreciation, two effects with opposite signs are at work: the external balance is improving, while additional inflation is imported with negative impacts on households real income. This inflationary effect is larger for Ghana, leading to an overall negative impact of a depreciation on the RGDP. However, the sign of the USD multiplier is highly dependent on parameters (in particular import and export elasticities), thus one should be cautious about the real effect of a depreciation on the RGDP.

[17] This is due to the definition of the consumer price index. Although the shares of the price of oil and of the imported goods in the definition of the CPI are the same in Ghana and in Mozambique, imports price end up weighting more in Ghana because they are indexes normalized in 1995, and the currency in Ghana depreciated more from 1995 to 2002, than in Mozambique, leading to higher indexes.

5 Conclusion

The aim of this paper is to present the detailed analytical structure of the AEO model which was originally developed by the Center of Development of the OECD to assist in the forecasting exercise of crucial macro variables for Africa. The simple analytics of the model are illustrated and briefly contrasted with those of other applied macro models such as the RMSM and the FP models. The second part of the paper derived analytical multipliers, aiming at further clarifying both the qualitative and quantitative properties of the model, as well as its limits. Numerical examples for Ghana and Mozambique complete this study.

6 Bibliography

Edward, S. (1989): *The international Monetary Fund and the Developing countries: A critical Evaluation*, Working Paper no. 2029, Cambridge, Mass.: National Bureau of Economic Research

Tarp, F. (1993) *Stabilization and Structural Adjustment – Macroeconomic Frameworks for Analysing the Crisis in sub-Saharan Africa*, London: Routledge

Tarp, F. and Brixen, P. (1996) *The South African Economy – Macroeconomic Prospects for the Medium Term, London: Routledge*

Taylor, L. (1988*) Varieties of Stabilization Experience: Toward Sensible Macroeconomics in the Third World*, Oxford: Clarendon Press

7 APPENDIX: List of variables and equations of the AEO model and correspondence with the Excel version

List of endogenous variables

Excel Name	Symbol	Description
	Real GDP Demand Side	
HhC	C_h	Household Consumption
GovCW	G	Government Real Current Expenditure
HhI	I_h	Private Investment

GovI	I_g	Public Investment (Volumes)
VStk	ΔS	Change in Stocks
Xmain	X_{main}	Main Exports
Xoth	X_{oth}	Other Exports
Xnfs	X_{nfs}	Non-Factor Services Exports (in real terms)
Mene	M_{ene}	Energy Imports
Moth	M_{oth}	Other Imports
Mnfs	M_{nfs}	Non-Factor Services Imports (in real terms)
RGDP	RGDP	Real GDP market prices
Price Indexes		
CPI	CPI	Consumer Price Index
Pinv	P_{inv}	Price Index Investment goods (incl. itax)
Px	P_X	Price X of other goods (LC incl. Taxes)
Pm_ene	P_M^{ene}	Price M energy (LC incl. Taxes)
Pm_oth	P_M^{oth}	Price M non energy (LC incl. Taxes)
Pd	P_d	Price Index domestic goods (incl. itax)
Pintm	P_{int}	Price Index Intermediates (incl. itax)
Pva	P_{va}	Price of Value Added
MargD	$Marg_D$	Margin Rate on domestic goods
Fiscal and Income accounts		
DtaxRev	Rev_{tax}^{dir}	Direct (Hh) Taxes Revenues
ItaxRev	Rev_{tax}^{ind}	Domestic Indirect Taxes Revenues
TdTaxRev	Rev_{tax}^{tar}	Trade taxes
RGDPpriv	$RGDP_{priv}$	RGDP Private Sector
Yhh_oth	Yh_{oth}	Household Non-agricultural Income
Yhh_food	Yh_{food}	Agriculture Income, Food
Yhh_agrx	Yh_{agrX}	Agriculture Income, Export crops
RealDY	Y_d^{real}	Real Disposable Income
GovCW_val	G^{val}	Government Consumption
GovI_val	I_g^{val}	Public Investment
OthTaxRev	Rev_{tax}^{oth}	Other taxes
		Other revenues
Grants	Grants	Grants
GovWbill	G_W	Wages and salaries
GovC_val	C_g^{val}	Government consumption of goods and services
GovOth_val	$C_g^{oth,val}$	Other current expenditure

GovTrHh	Tr_h^g	Current transfers
IntPaymt	Int_g	Interest payments
faGovI_val	$I_g^{fa,val}$	Capital expenditure
NetLend	NetLend	Net lendings
FaGovCW_val	$G^{fa,val}$	Current Gov. Expenditure (Fiscal Acc.t)
DomD	D_{dom}	Domestic Demand
Miscellaneous		
WPm_ene	WP_M^{ene}	World Price for M energy (USD)
WP_EUR	WP^{EUR}	World Prices for EURO zone
WP_ROW	WP^{ROW}	World Prices for ROW zone
Pfood	P_{food}	Food Price Index
Ppub	P_{pub}	Public Utilities Price Index
GovWIndex	W_{pub}	Government Wage Index
PrivW	W_{priv}	Private Wages Index
REm_nene	RE_M	Effective Real Exch Rate Import pm/pd
Rex	RE_X	Effective Real Exch Rate Export px/pd
EUR	EUR	LC per EURO
Wdem	D_{World}	World Demand Index
GDP_fc	GDP_{fc}	GDP at factor prices
Mene_val	M_{ene}^{val}	Energy Imports current value
Moth_val	M_{oth}^{val}	Other Imports current value
Mnfs_val	M_{nfs}^{val}	Non-Factor Services Imports current value
Xmain_val	X_{main}^{val}	Main Exports
Xoth_val	X_{oth}^{val}	Other Exports
Xnfs_val	X_{nfs}^{val}	Non-Factor Services Exports
HhC_val	C_h^{val}	Households Consumption
HhI_val	I_h^{val}	Private Investment
VStk_val	ΔS^{val}	Change in Stocks

List of exogenous variables

Income and Fiscal Block		
gPrivW	gW_{priv}	Growth Rate Private Wages
gYhh_food	gYh_{food}	Food Production growth rate
gPp_food	gP_{prod}^{food}	Food Producer prices change
gGovEmpl	$gEmpl_{pub}$	Government Employment growth
gGovW	gW_{pub}	Government Wage growth

gGovExp	gC_g	Growth rate of Real Gov. Expend (excl. wages)
gGovI	gI_g	Growth rate of Real Gov. Invest.t
RowTrHh	Tr_h^{ROW}	ROW transfers to Hh
Export Block		
Xs1_vol	X_1	Export Sector 1 in volume
Xs2_vol	X_2	Export Sector 2 in volume
Xs3_vol	X_3	Export Sector 3 in volume
Xs4_vol	X_4	Export Sector 4 in volume
WPX1	WP_X^1	World Price of export sector 1 (USD)
WPX2	WP_X^2	World Price of export sector 2 (USD)
WPX3	WP_X^3	World Price of export sector 3 (USD)
WPX4	WP_X^4	World Price of export sector 4 (USD)
PpX1	$P_{X,prod}^1$	Producer Price of export sector 1 (LC)
PpX2	$P_{X,prod}^2$	Producer Price of export sector 2 (LC)
PpX3	$P_{X,prod}^3$	Producer Price of export sector 3 (LC)
PpX4	$P_{X,prod}^4$	Producer Price of export sector 4 (LC)
gWDem	gD_{world}	World Demand growth rate
gPop	gPop	Population growth rate
ghhI	gI_h	Growth rate of Private Investment
EUR	EUR	LC per EURO
USD	USD	LC per USD
EurUsd	EUR_{USD}	USD per EURO
Poil	P_{oil}	Oil Price USD per bbl
gP_EUR	gP_{EUR}	Inflation Euro zone
gP_ROW	gP_{ROW}	Inflation ROW zone
gP_pub	gP_{pub}	Public Utilities prices change

List of parameters

shrC_Doth	$\sigma_C^{dom,oth}$	Share of Domestic other goods in Consumption[18]
shrC_Dfood	$\sigma_C^{dom,food}$	Share of Domestic Food in Consumption
shrC_M	σ_C^M	Share of imported goods in Consumption

[18] In these shares, consumption mean final consumption excluding auto-consumption of agricultural products: the shares are used to generated the Consumer Price Index which cannot include prices of products that are not sold.

shrC_pub	σ_C^{pub}	Share of public utilities in Consumption
alphaC	α_C	Average propensity to consume
shr_intm	σ_{int}	Share of Intermediates in Production
shr_Dintm	σ_{int}^{dom}	Share of Domestic Intermediates
shr_MOintm	$\sigma_M^{oil,int}$	Share of M Intermediates oil
shr_MnOintm	$\sigma_M^{noil,int}$	Share of M Intermediates non-oil
Tech	τ	Technological Progress rate
ElaL	ε_L	Elasticity of Labor[19]
shrX_EUR	σ_X^{EUR}	Share of Exports From EURO zone
shrX_ROW	σ_X^{ROW}	Share of Exports From ROW
shrM_EUR	σ_X^{EUR}	Share of Imports From EURO zone
shrM_ROW	σ_M^{ROW}	Share of Imports From ROW
shr_Minv	σ_M^{inv}	Imported share of Investment goods
margM	μ_M	Margin rates on imports
adjGI	Adj_{Ig}	Adjusting Ratio for Gov. Investment
adjGCW	Adj_G	Adjusting Ratio for Gov. current expenditure
dirtxRate	r_{tax}^{dir}	Domestic direct Tax rate
itxRate	r_{tax}^{ind}	Domestic Indirect Tax rate
TarRate	r_{tax}^{tar}	Tariff Rate
hystC	χ_C	Hysteresis parameter
YSpeed	ζ_Y	Private Income Speed of adjustment[20]
MSpeed	ζ_M	Imports Speed of adjustment
ElaYM	ε_Y^M	Imports Income-elasticity
ElaPrM	ε_P^M	Imports price-elasticity
XSpeed	ζ_X	Exports Speed of adjustment

[19] To be more precise, this is elasticity of production to labor.

[20] To be more precise, this is the speed of adjustment of non agricultural, private, formal and informal employment.

ElaYX	ε_{Y}^{X}	Exports Income-elasticity[21]
ElaPrX	ε_{P}^{X}	Exports price-elasticity

List of equations

	Real GDP Demand Side	
HhC	$C_h = \chi_C \cdot C_{h\_1} \cdot (1+gPop) + (1-\chi_C) \cdot \alpha_C \cdot Y_d^{real}$	Household Consumption
GovCW	$G = adj_G \cdot \left(\frac{C_g^{val} + C_g^{oth,val}}{P_{inv}} + \frac{G_W}{W_{pub}} \right)$	Government Real Current Expenditure
HhI	$I_h = I_{h\_1} \cdot (1+gI_h)$	Private Investment
GovI	$I_g = \frac{I_g^{val}}{P_{inv}}$	Public Investment (Volumes)
VStk	$\Delta S = \Delta S\_1$	Change in Stocks
Xmain	$X_{main} = \sum_{i=1}^{4} X_i \cdot WP_X^{\,i}\_base \cdot USD\_base$	Main Exports
Xoth	$X_{oth} = X_{oth\_1} \cdot \left(\frac{X_{oth\_1}}{X_{oth\_2}} \right)^{1-\varsigma_X} \cdot \left(\frac{D_{world}}{D_{world\_1}} \right)^{\varepsilon_Y^X \varsigma_X} \cdot \left(\frac{RE_X}{RE_{X\_1}} \right)^{\varepsilon_P^X \varsigma_X}$	Other Exports
Xnfs	$X_{nfs} = X_{nfs\_1} \cdot \frac{X_{oth}}{X_{oth\_1}}$	Non-Factor Services Exports (in real terms)
Mene	$M_{ene} = M_{ene\_1} \cdot \left(\frac{M_{ene\_1}}{M_{ene\_2}} \right)^{1-\varsigma_M} \cdot \left(\frac{D_{dom}}{D_{dom\_1}} \right)^{\varepsilon_Y^M \varsigma_M}$	Energy Imports
Moth	$M_{oth} = M_{oth\_1} \cdot \left(\frac{M_{oth\_1}}{X_{oth\_2}} \right)^{1-\varsigma_M} \cdot \left(\frac{D_{dom}}{D_{dom\_1}} \right)^{\varepsilon_Y^M \varsigma_M} \cdot \left(\frac{RE_M}{RE_{M\_1}} \right)^{\varepsilon_P^M \varsigma_M}$	Other Imports
Mnfs	$M_{nfs} = M_{nfs\_1} \cdot \frac{M_{oth}}{M_{oth\_1}}$	Non-Factor Services Imports (in real terms)
RGDP	$RGDP = C_h + G + I_h + I_g + \Delta S + (X_{main} + X_{oth} + X_{nfs}) - (M_{ene} + M_{oth} + M_{nfs})$	Real GDP market prices
	Price Indexes	
CPI	$CPI = (1+Marg_D) \cdot \left(\sigma_C^{dom,food} \cdot P_{food} + \sigma_C^{dom,oth} \cdot P_d + \sigma_C^{M} \cdot \left(\frac{1+\mu_M}{1+\mu_{M\_base}} \right) \cdot P_M^{oth} + \sigma_C^{pub} \cdot P_{pub} \right)$	Consumer Price Index

[21] To be more precise, this is elasticity of exports to world demand.

Pinv	$P_{inv} = \sigma_M^{inv} \cdot P_M^{oth} + \left(1 - \sigma_M^{inv}\right) \cdot P_d$	Price Index Investment goods (incl. itax)
Px	$P_X = \sigma_X^{EUR} \cdot \frac{EUR}{EUR\_base} WP^{EUR} + \sigma_X^{ROW} \cdot \frac{USD}{USD\_base} WP^{ROW}$	Price X of other goods (LC incl. Taxes)
Pm_ene	$P_M^{ene} = WP_M^{ene} \cdot \frac{USD}{USD\_base} \cdot \frac{1 + r_{tax}^{tar}}{1 + r_{tax}^{tar}\_base}$	Price M energy (LC incl. Taxes)
Pm_oth	$P_M^{oth} = \left(\sigma_M^{EUR} \cdot \frac{EUR}{EUR\_base} WP^{EUR} + \sigma_M^{ROW} \cdot \frac{USD}{USD\_base} WP^{ROW} \right) \cdot \frac{1 + r_{tax}^{tar}}{1 + r_{tax}^{tar}\_base}$	Price M non energy (LC incl. Taxes)
Pd	$P_d = \left[\sigma_{int} \cdot P_{int} + \left(1 - \sigma_{int}\right) \cdot P_{va} \right] \cdot \frac{1 + r_{tax}^{ind}}{1 + r_{tax}^{ind}\_base}$	Price Index domestic goods (incl. itax)
Pintm	$P_{int} = \sigma_{int}^{dom} \cdot P_d + \sigma_M^{oil,int} \cdot P_M^{ene} + \sigma_M^{noil,int} \cdot P_M^{oth}$	Price Index Intermediates (incl. itax)
Pva	$P_{va} = W_{priv} \cdot e^{-\frac{\tau \cdot (y - y\_base)}{\varepsilon_L}} \cdot \left(\frac{RGDP_{priv}}{RGDP_{priv}\_base} \right)^{\frac{1}{\varepsilon_L} - 1}$	Price of Value Added
margD	$Marg_D = Marg_D\_1$	Margin Rate on domestic goods
Fiscal and Income accounts		
DTaxRev	$Rev_{tax}^{dir} = r_{tax}^{dir} \cdot \left(Yh_{oth} + G_W \right)$	Direct (Hh) Taxes Revenues
ItaxRev	$Rev_{tax}^{ind} = r_{tax}^{ind} \cdot GDP_{fc}$	Domestic Indirect Taxes Revenues
TdTaxRev	$Rev_{tax}^{tar} = r_{tax}^{tar} \cdot \left(M_{ene}^{val} + M_{oth}^{val} \right)$	Trade taxes
RGDPpriv	$RGDP_{priv} = RGDP - \frac{adj_G}{W_{pub}} \cdot G_W - X_{main} - \frac{Yh_{food}}{P_{food}}$	RGDP Private Sector
Yhh_oth	$Yh_{oth} = Yh_{oth}\_1 \cdot \left(1 + gW_{priv}\right)\left(1 + gW_{priv}\_1\right)^{\xi_Y - 1} \cdot \left(\frac{Yh_{oth}\_1}{Yh_{oth}\_2} \right)^{1 - \xi_Y} \cdot \left(\frac{RGDP_{priv}}{RGDP_{priv}\_1} \cdot e^{-\tau} \right)^{\frac{\xi_Y}{\varepsilon_L}}$	Household Non-agricultural Income
Yhh_food	$Yh_{food} = Yh_{food}\_1 \cdot \left(1 + gYh_{food}\right)\left(1 + gP_{prod}^{food}\right)$	Agriculture Income, Food
Yhh_agrx	$Y_h^{agr,X} = \sum_{i=1}^{4} X_i \cdot P_{X,prod}^{\ i}$	Agriculture Income, Export crops
RealDY	$Y_d^{real} = \frac{Yh_{food} + Yh_{agr,X} + Yh_{oth} + G_W + Tr_h^g + Tr_h^{ROW} - Re}{CPI}$	Real Disposable Income

GovCW_val	$G^{val} = adj_G \left(G_W + C_g^{val} + C_g^{oth,val} \right)$	Government Consumption
GovI_val	$I_g^{val} = adj_{I_g} \cdot I_g^{fa,val}$	Public Investment
OthTaxRev	$Rev_{tax}^{oth} = Rev_{tax}^{oth}\_1 \cdot \frac{GDP}{GDP\_1}$	Other taxes
OthRev	$Rev_{oth} = Rev_{oth}\_1 \cdot \frac{GDP}{GDP\_1}$	Other revenues
GovWbill	$G_W = G_W\_1 \cdot (1 + gEmpl_{pub})(1 + gW_{pub})$	Wages and salaries
GovC_val	$C_g^{val} = C_g^{val}\_1 \cdot (1 + gC_g) \cdot \frac{P_{int}}{P_{int}\_1}$	Other purchases of goods & services
GovOth_val	$C_g^{oth,val} = C_g^{oth,val}\_1 \cdot (1 + gC_g) \cdot \frac{P_{int}}{P_{int}\_1}$	Other current expenditure
GovTrHh	$Tr_h^g = Tr_h^g\_1 \cdot (1 + gC_g) \cdot \frac{CPI}{CPI\_1}$	Current transfers
IntPaymt	Int_g	Interest payments
faGovI_val	$I_g^{fa,val} = I_{gov}^{fa,val}\_1 \cdot (1 + gI_g) \cdot \frac{P_{inv}}{P_{inv}\_1}$	Capital expenditure
NetLend	NetLend	Net lendings
faGovCW_val	$G^{fa,val} = G_W + C_g^{val} + C_g^{oth,val}$	Current Gov. Expenditure (Fiscal Acc.t)
Miscellaneous		
DomD	$D_{dom} = C_h + G + I_h + I_g + \Delta S - adj_G \cdot \frac{G_W}{W_{pub}}$	Domestic Demand[22]
WPm_ene	$WP_M^{ene} = WP_M^{ene}\_1 \cdot \frac{P_{oil}}{P_{oil}\_1}$	World Price for M energy (USD)
WP_EUR	$WP^{EUR} = WP^{EUR}\_1 \cdot (1 + gP_{EUR})$	World Prices for EURO zone
WP_ROW	$WP^{ROW} = WP^{ROW}\_1 \cdot (1 + gP_{ROW})$	World Prices for ROW zone
Pfood	$P_{food} = P_{food}\_1 \cdot (1 + gP_{prod}^{food})$	Food Price Index
Ppub	$P_{pub} = P_{pub}\_1 \cdot (1 + gP_{pub})$	Public Utilities Price Index
GovWIndex	$W_{pub} = W_{pub}\_1 \cdot (1 + gW_{pub})$	Government Wage Index
PrivW	$W_{priv} = W_{priv}\_1 \cdot (1 + gW_{priv})$	Private Wages Index
REm_nene	$RE_M = \frac{P_M^{oth}}{P_d}$	Effective Real Exch Rate Import pm/pd
REx	$RE_X = \frac{P_X}{P_d}$	Effective Real Exch Rate Export px/pd

[22] Cf. note 16

EUR	$EUR = USD \cdot EUR_{USD}$	LC per EURO
WDem	$D_{world} = D_{world\_1} \cdot (1 + gD_{world})$	World Demand Index
GDP_fc	$GDP_{fc} = C_h^{val} + G^{val} + I_h^{val} + I_g^{val} + \Delta S^{val} + (X_{main}^{val} + X_{oth}^{val} + X_{nfs}^{val}) - (M_{ene}^{val} + M_{oth}^{val} + M_{nfs}^{val}) - Rev_{tax}^{ind} - Rev_{tax}^{tar}$	GDP at factor prices
Mene_val	$M_{ene}^{val} = M_{ene} \cdot P_M^{ene} \cdot \frac{1 + r_{tax\_base}^{tar}}{1 + r_{tax}^{tar}}$	Energy Imports current value
Moth_val	$M_{oth}^{val} = M_{oth} \cdot P_M^{oth} \cdot \frac{1 + r_{tax\_base}^{tar}}{1 + r_{tax}^{tar}}$	Other Imports current value
Mnfs_val	$M_{nfs}^{val} = M_{nfs} \cdot P_M^{oth}$	Non-Factor Services Imports current value
Xmain_val	$X_{main}^{val} = \sum_{i=1}^{4} X_i \cdot WP_X^{\ i} \cdot USD$	Main Exports
Xoth_val	$X_{oth}^{val} = X_{oth} \cdot P_X$	Other Exports
Xnfs_val	$X_{nfs}^{val} = X_{nfs} \cdot P_X$	Non-Factor Services Exports
HhC_val	$C_h^{val} = C_h \cdot CPI$	Households Consumption
HhI_val	$I_h^{val} = I_h \cdot P_{inv}$	Private Investment
VStk_val	$\Delta S^{val} = \Delta S \cdot P_{inv}$	Change in Stocks